NAME: Soln

Test Prep 2

There are two quick solving problems on this page. If you finish this page, try the extra problems on the back (for an extra point). You have 10 minutes!

1. (Separable) Find the general explicit solution to $\frac{1}{t}\frac{dy}{dt} = y^2 e^{t^2} + e^{t^2}$.

$$\frac{1}{t}\frac{dy}{dt} = (y^2+1)e^{t^2}$$

$$\int \frac{1}{y^2+1}\,dy = \int t\,e^{t^2}\,dt$$

$u = t^2$
$du = 2t\,dt$
$\frac{1}{2t}du = dt$

$$\tan^{-1}(y) = \int e^u \frac{1}{2}\,du$$

$$\tan^{-1}(y) = \frac{1}{2}e^u + C$$

$$\boxed{y = \tan\left(\frac{1}{2}e^{t^2} + C\right)}$$

2. (Integrating Factor) Find the general explicit solution to $\frac{dy}{dt} + \frac{2}{t}y = t^3$ (You may assume $t > 0$).

$$\int \frac{2}{t}\,dt = 2\ln(t) + C$$ ← I choose $C = 0$

$$\mu(t) = e^{2\ln(t)} = e^{\ln(t^2)} = t^2$$

$$t^2\frac{dy}{dt} + 2ty = t^5$$

$$\frac{d}{dt}(t^2 y) = t^5$$

$$t^2 y = \frac{1}{6}t^6 + C$$

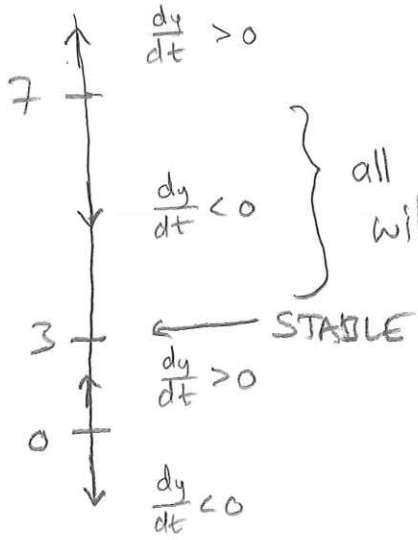
$$\boxed{y = \frac{1}{6}t^4 + \frac{C}{t^2}}$$

Extra Credit Problem: Consider $\frac{dy}{dt} = y^3 - 10y^2 + 21y$.
If $y = f(t)$ is a solution to the differential equation with the initial condition $y(1) = 5$, then what is $\lim_{t\to\infty} f(t)$?

Explain your answer.
(Hint: You will NOT have to integrate anything to answer this questions).

$\frac{dy}{dt} = y(y^2 - 10y + 21) = y(y-3)(y-7)$

The equilibrium sol'ns are $y=0$, $y=3$, $y=7$



all sol'ns with initial conditions here will decrease and approach $y=3$

Since $y(1) = 5$ and 5 is between 3 & 7

we have

$$\lim_{t\to\infty} f(t) = 3$$

Do you have a specific or general question about the course so far (optional)? If so, feel free to write it here and hopefully I can answer your question in an upcoming lecture:

LET ME KNOW!